Glassy dynamics near zero temperature

Federico Ricci-Tersenghi† and Riccardo Zecchina‡
The Abdus Salam International Center for Theoretical Physics, Condensed Matter Group
Strada Costiera 11, P.O. Box 586, I-34100 Trieste, Italy

We numerically study finite-dimensional spin glasses at low and zero temperature, finding evidences for (i) strong time/space heterogeneities, (ii) spontaneous time scale separation and (iii) power law distributions of flipping times. Using zero temperature dynamics we study blocking, clustering and persistence phenomena.

The study of non-equilibrium dynamics in quenched spin systems provides a rigorous and numerical test ground for modeling glassy behavior. Among the open issues which arise in this context there is the connection between spatial and temporal heterogeneities and the global off-equilibrium dynamical features characteristic of glassy structures such as aging [1]. In the last decade, much progress has been made in the study of domain evolution and aging in both ordered and disordered systems [2–4]. In particular, the mean-field picture of the glassy transition [5,6] plays a central role in interpreting results of numerical simulations on spin systems, but it can not account for heterogeneities. On the other hand a set of interesting numerical and analytical results on the off-equilibrium dynamics in non-frustrated systems have been obtained [7–9], which can be now extended to frustrated ones.

The scope of this study is to provide a further link between the different aspects of the glassy transition unfolded by the various approaches. More specifically, we shall focus on models with discrete ($\pm J$) couplings in 2 and 3 dimensions. Of particular interest is the onset of time scale separation in the low temperature (low T) regime and its connections with the blocking and persistence phenomena in the zero-temperature ($T=0$) limit.

The choice of discrete couplings stems from the fact that they naturally arise as interactions incorporating geometrical frustration and constraints. Moreover, models with discrete couplings are of central relevance in other areas such as combinatorial optimization and are known to provide very rich dynamical features even at $T=0$ [10].

In what follows, we report results from extensive numerical simulations concerning the following issues.

I) The onset and the dependence on waiting time of the spontaneous time scale separation and space clustering at sufficiently low T in 2D and 3D models.

II) The related emergence at $T=0$ of the "blocking" phenomenon together with the clustering of a finite fraction of spins that flip infinitely often ("fast" spins), along with its interpretation in terms of the functional mean-field order parameter.

III) The $T=0$ distributions of flipping times and fast spins clusters sizes.

IV) The persistence phenomenon.

Our studies focus on the $\pm J$ Edwards-Anderson (EA) model in 2 and 3 spatial dimensions (square and cubic lattices). The Hamiltonian of these models reads

$$\mathcal{H} = \sum_{(i,j)\in E} s_i J_{ij} s_j \qquad , \tag{1}$$

where E is the set of lattice edges, the spins are of Ising type and the couplings take the values ± 1 with equal probability.

The study of 3D EA model in the very low temperature region requires huge thermalization times and therefore it has been severely limited by available computer resources. Only very recently these studies have been pushed to very low temperatures, thanks to the use of parallel computers [6]. The data presented in this letter referring to the 3D case has been obtained with the help of the APE100 parallel computer [11]. The 2D case is indeed much simpler in virtue of the existence of polynomial algorithms for ground-states calculations.

It is known that in both 2D and 3D EA models [12] lowering temperature produces a surprising increase in the number of high frequency flipping spins. Here we push the study of the flipping times distribution to the very low temperature region in the hard case of the 3D EA model. Moreover we investigate the dependence of this distribution on the waiting time, which is a relevant feature of the aging regime in glassy phases.

According to the typical scheme used in off-equilibrium dynamical studies [6], we simulate large systems (of at least 32^3 spins) and we start the experiment with an instantaneous quench from infinite temperature to one in the glassy phase ($T < T_c \simeq 1.1$). Next we let the system evolve for a waiting time t_w (times are expressed in terms of Monte Carlo sweeps, MCS). Finally we calculate the flipping rates probability distribution function (pdf) measuring the number of flips done by every spin within time windows extending from $t_w + t$ to $t_w + 2t$, where $t = 2^k$ (with $k \leq 26$) is also the time window size. We simply define the mean flipping time τ as the time window size divided by the number of flips and we construct the pdf of its logarithm, $P_{t,t_w}(\ln \tau)$, by taking the histogram over all spins (with the label t identifying the time window). This distribution is expected to be self-averaging (like correlation functions) and so we prefer to simulate few very large samples. The choice of working with the $\ln(\tau)$ pdf instead of that for τ is dictated by the

broadness of the latter. Note that a pure exponential tail in the former $[P(\ln\tau) \propto e^{-\lambda \ln\tau}]$ corresponds to a power law tail in the latter $[P(\tau) \propto \tau^{-\lambda-1}]$.

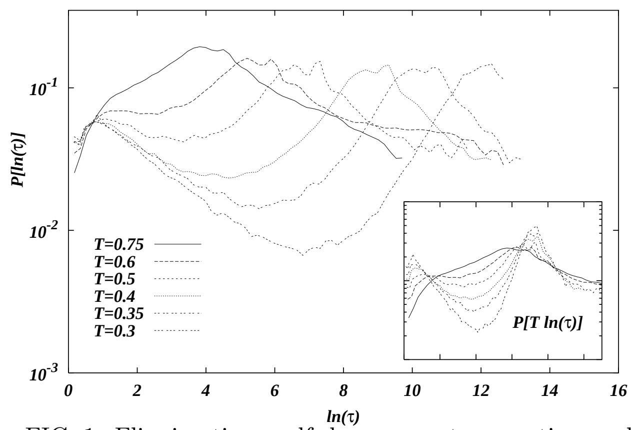

FIG. 1. Flipping times pdf show a spontaneous time scales separation as temperature is decreased. In the inset we present the pdf of the scaling variable $T\ln(\tau)$

In the $t_w = 0$ case we find, as expected, that $P_{t,0}(\ln\tau)$ does not depend on t and so we consider only the pdf measured during the last time window, which has the largest support (more than 10^6 MCS) and the highest statistics. In Fig. 1 we show such a distribution for different temperatures. While at very high temperatures the pdf is strongly peaked around the mean, it acquires very large tails approaching the glass transition temperature [12] and finally develops a clear bimodal shape at very low temperatures, as can be seen in Fig. 1 (note the logarithmic scale on the y axis). The emergence of a spontaneous time scales separation, allows us to naturally divide the spins into two very general classes: the "fast" spins belonging to the left peak and the "slow" ones to the right peak. While the shape of the left part of the distribution does not change with temperature, the position of the "slow" peak is clearly moving towards higher values when the temperature is decreased. As shown in the inset of Fig. 1, this process verify the simple scaling $T\ln(\tau)$, commonly found in every activated process in spin glasses. It follows that slow spins are the ones that have to overcome a barrier in order to flip, while the fast ones will eventually have a zero local field at some time (which could even happen very rarely). In the zero temperature limit we expect that the slow peak moves towards unreachable time-scales, whereas fast spins are the only responsible for the dynamics. The adjective "fast" could be somehow misleading, in turn their actual flipping times follow a very broad distribution.

It is well known that in the aging regime two-times quantities depend on both times and not only on their difference. In the case of our $P_{t,t_w}(\ln\tau)$ this dependence on the waiting time is shown in Fig. 2, where we present data for a low temperature ($T = 0.35$), a very large waiting time ($t_w = 2^{24}$) and many values of t. During the aging process two different regimes can be identified [3,13]: the *quasi-equilibrium* regime ($t \ll t_w$) where the system relaxes inside a quasi-state and the *aging* regime ($t \geq t_w$) where macroscopic rearrangements take place. In these two regimes the shape of the $P_{t,t_w}(\ln\tau)$ is different (this has been verified in many simulations with different t_w values, even if here we report the results concerning only one waiting time). In Fig. 2 we display the pdf for different choices of the ratio t/t_w: the lowest curves correspond to the quasi-equilibrium regime, while the upper ones have values of t of the order of t_w and therefore include the effects coming from the aging regime.

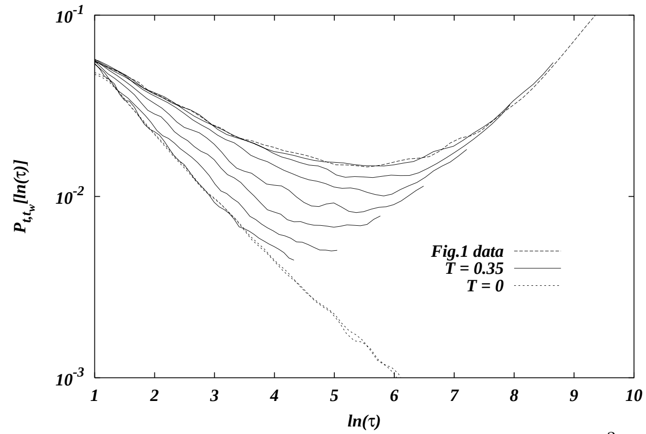

FIG. 2. $T = 0.35$ data has been measured on a 32^3 system (10 samples) after waiting 2^{24} MCS. Different curves correspond to different time window sizes (from bottom) $t = 2^{12}, 2^{14}, 2^{16}, 2^{18}, 2^{20}, 2^{22}, 2^{24}$. The last curve coincides with data presented in Fig.1. $T = 0$ data have been measured on 100^3 systems, after a waiting time of $t_w = 2^{16}, 2^{18}$.

It is clear that the time-scales separation is sharper in the quasi-equilibrium regime, while approaching the aging regime the gap between fast and slow peaks is partially filled. Observing that the slow spins create the environment where the fast spins move in (the so called cage effect), we may assume that the meta-stable state is identified by the spatial structure of slow spins which act as a rigid backbone on time scales typical of the quasi-equilibrium regime. Therefore we may also observe that the basic excitations that bring the system out of quasi-equilibrium are given by those spins which fill the gap between the peaks. The study of the spatial structure of these spins would help towards the understanding of basic excitation in 3D spin glasses.

The same set of simulations with Gaussian couplings gives no evidence of a so simple time scale separation. In this case the distribution of microscopic time scales is different and much longer simulations would be needed in order to unveil the separation. Moreover at $T = 0$ in the Gaussian case the system rapidly get stuck in a local minimum and its configuration is fully frozen, con-

sistently with the zero-temperature classification given in [9]. Only the $\pm J$ model has a rich aging behavior at zero temperature.

In the $T = 0$ limit activated processes disappear and the slow peak goes to infinity. Then the gap between the peaks can no longer be filled when the system leaves the quasi-equilibrium regime. In other words, the $P_{t,t_w}(\ln\tau)$ becomes t_w-independent (we have checked numerically this fact) and it describes only the fast spin component. Remarkably enough, the $T = 0$ pdf is already present in the finite temperature data, in their quasi-equilibrium regime (see Fig. 2) and this would suggest that $T = 0$ dynamical features could be present at low temperatures too, at least in the quasi-equilibrium regime. Hereafter we will consider only the following updating rule. A site is randomly chosen and it is oriented in the direction of the neighbors majority or at random if the local field on it is zero.

We find that the $T = 0$ flipping time distributions can be well fitted by a power law over many decades, with an exponent $\lambda + 1 = 1.76(2)$ in 3D and $\lambda + 1 = 2.08(3)$ in 2D. This means that in 2D after a large but *finite* time (given by the average flipping time) an extensive fraction of the system can be flipped with zero-energy costs. On the contrary in 3D an average flipping time can not be defined and this divergence is maybe related to the existence of a finite temperature phase transition.

Persistence [7] is one the most studied properties of zero temperature dynamics [8,9], nevertheless it has been never measured in spin glasses. Here we fill this gap.

The persistence $U(t, t_w)$ is defined as the number of unflipped spins in the time interval $[t_w, t_w + t]$. In pure ferromagnetic models (at least for $D = 2, 3$) the persistence $U(t, t_w)$ decays to zero, in the large times limit, as $U(t, t_w) \propto (t/t_w)^{-\theta(D)}$, the exponent being t_w-independent. On the contrary, in disordered models the persistence is expected to remain finite [9] in the $t \to \infty$ limit and its asymptotic value $U_\infty(t_w)$ may depend on the waiting time. The physical mechanism underlying such effect is the slow freezing of a rigid component. It follows that the proper way of estimating the persistence in disordered systems is by taking a sufficiently large value of t_w, differently from what has been done in previous studies where the choice $t_w = 0$ was used.

In Fig. 3 we show the results for 3D $\pm J$ spin glasses (very similar results have been found in the 2D case). While the system relaxes towards the stationary state the number of frozen spins grows and $U_\infty(t_w)$ monotonically increases with t_w. In the large times limit we can extract the θ exponent from the decay $U(t, t_w) - U_\infty(t_w) \propto t^{-\theta}$. This fitting procedure is not an easy one, because of the slow persistence decay, and so our results are affected by large errors. Our best estimations are $\theta = 0.46(3)$ and $U_\infty(\infty) = 0.862(2)$ in 3D and $\theta = 0.64(3)$ and $U_\infty(\infty) = 0.778(2)$ in 2D. The θ value in 3D is confirmed by the local magnetizations pdf which can be well fitted by the law $(1 - m^2)^{\theta-1}$ at low and zero temperatures.

These numbers imply that in the large times limit a $\pm J$ spin glass is not completely frozen. While a large fraction $U_\infty(\infty)$ of the spins get blocked forever, a *finite* fraction $1 - U_\infty(\infty)$ of the volume is composed by spins that flip infinitely often [14], with their flipping times following power law distributions. The fast spins produce the persistence phenomenon.

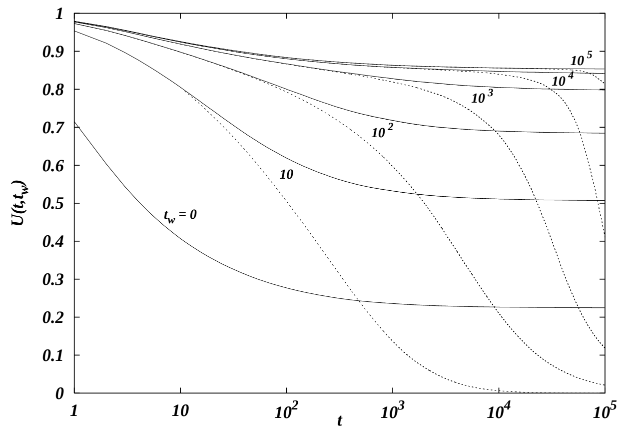

FIG. 3. Continuous lines show the persistence decay after different waiting times in a 100^3 spin glass. Dashed lines present the persistence in a microcanonical experiment, where the energy is kept constant after time t_w.

In Fig. 3 we report with dashed lines the results of a microcanonical experiment. As before, we let the system relax for t_w MCS and then we measure the persistence. The difference is that now the dynamics after time t_w is performed keeping the energy constant to the value it had at time t_w. For $t \le t_w$ the data perfectly coincide with those for the usual persistence. Then the system leaves the configurational space region (state [6]) where it was confined by the dynamics and it goes far away (we check this also measuring correlations functions, that behave qualitatively like persistence). Thus we can conclude that the freezing phenomenon is completely dynamical [15]. In other words the system seems to be stuck only because it always finds a path towards a lower energy configuration belonging to the same state before finding a path towards a different state.

For both dimensionalities we find that fast spins are organized in clusters. At any time and in each cluster there must be at least one "free" spin that can be flipped without any energy cost ($\Delta E \le 0$). The number of free spins equals the number of flat directions in the configuration space and decreases lowering the energy. Free spins usually appear in pairs, which wander inside their cluster [16].

Fast spins clusters sizes follow the pdf shown in Fig. 4. Measurements have been taken after a waiting time of $t_w = 10^4$. Note that a finite t_w can overestimate a little the clusters size, however for both dimensions we have

that $U_\infty(10^4)$ is very near to $U_\infty(\infty)$ and we did not find any dependence on t_w. Up to our accuracy, 3D data can be well fitted by a power law with an exponent $-1.94(3)$. On the contrary 2D data clearly show a cut-off. We can not exclude that also 3D data would show a cut-off on larger scales.

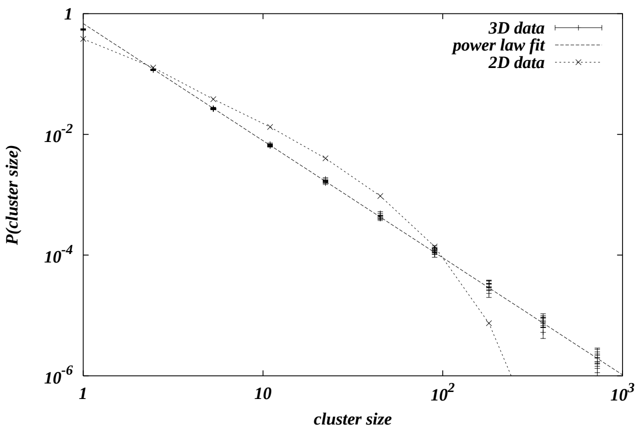

FIG. 4. Fast spins clusters sizes pdf. 3D data have been collected in system of sizes 30^3, 45^3 and 100^3, and with two different updating rules (sequential and random). The power law fit gives $-1.94(3)$ as the best exponent. 2D data, measured in a 1000^2 system, clearly show a cut-off.

For any finite cluster we can construct the set of allowed configurations and then we can evaluate the time a random walker needs to visit all of them. This time is, by construction, greater than the larger flipping time of any spin in the cluster. In this way we can link clusters sizes and flipping times, and one would expect to find a cut-off τ_{max} in the 2D distribution of flipping times as a consequence of the one present at c_{max} in Fig. 4. However times may be exponential in the cluster size (like e.g. the recurrence time) and τ_{max} may be considered infinite for all practical purposes. Moreover, if one assume from percolation arguments that c_{max} grows logarithmically with the system size N, then τ_{max} would grow as a power of N, like usually in any finite size systems.

In conclusion we have performed a numerical study of the low temperature dynamics of a finite-dimensional frustrated model with discrete couplings, namely the EA model in 2D and 3D, which can be viewed as a prototype model for glassy systems. The onset of a spontaneous time/space scale separation allows for a classification of spins in distinct classes characterized by their own dynamics as well as the identification of low-energy excitation responsible for structural rearrangements during the aging.

In the $T=0$ limit, while a part of the system completely freeze, the fast spins dynamics can be quantitatively described in terms of power law exponents which distinguish the 2D from the 3D case. The dynamics inside a cluster of fast spins, although diffusive-like, has strong constraints and can be interpreted along the lines of [17].

Given a proper definition of persistence in aging systems, we have been able to estimate the persistence exponent in the EA model, which turns out to be of kind $\mathcal{M}$ [9], i.e. partially frozen and partially infinitely flipping. Finally, with a microcanonical experiment, we have shown that blocking phenomena are completely dynamical, even in disordered systems.

In spite of the central role played by space and time heterogeneities in glassy dynamics, such aspects have been rather poorly studied from the analytical point of view. Scope of this paper has been to provide some reference numerical result which hopefully could be captured by theoretical arguments. A first step in this direction can be done within the framework of finitely connected long range models (like e.g. the Viana-Bray one [18]). In these models the underlying graph generates non trivial heterogeneities, driven by the connectivity pattern.

---

† E-mail: `riccife@ictp.trieste.it`

‡ E-mail: `zecchina@ictp.trieste.it`

[1] *Spin Glasses and Random Fields*, edited by A.P. Young. World Scientific (Singapore 1997).

[2] A. J. Bray, Adv. Phys. **43**, 357 (1994).

[3] J.P. Bouchaud, L.F. Cugliandolo, J. Kurchan and M. Mézard in [1], pag. 161.

[4] H. Rieger, Ann. Rev. Comp. Phys. **2**, 295 (1995).

[5] M. Mézard, G. Parisi, M. A. Virasoro, *Spin glass theory and beyond* (World Scientific, 1987).

[6] E. Marinari, G. Parisi, F. Ricci-Tersenghi, J.J. Ruiz-Lorenzo and F. Zuliani, J. Stat. Phys. **98**, 973 (2000).

[7] A.J. Bray, B. Derrida and C. Godrèche, Europhys. Lett. **27**, 175 (1994); J. Phys. A **27**, L357 (1994).

[8] B. Derrida, V. Hakim and V. Pasquier, Phys. Rev. Lett. **75**, 751 (1995); J. Stat. Phys. **85**, 763 (1996).

[9] C.M. Newman and D.L. Stein, Phys. Rev. Lett. **82**, 3944 (1999); Physica A **279**, 159 (2000).

[10] A. Barrat and R. Zecchina, Phys. Rev. E **59**, 1299 (1999).

[11] C. Battista *et al.*, Int. J. High Speed Comp. **5**, 637 (1993).

[12] P.H. Poole, S.C. Glotzer, A. Coniglio and N. Jan, Phys. Rev. Lett. **78**, 3394 (1997).

[13] S. Franz and M.A. Virasoro, J. Phys. A **33**, 891 (2000).

[14] The same happens in a 2D spin glass too. For a proof see A. Gandolfi, C.M. Newman and D.L. Stein, to appear in Communications in Mathematical Physics (2000).

[15] This conclusion is trivial for non-disordered models where domain walls can freely wander, but it was not so for disordered models where domain walls are pinned.

[16] The movement of a pair is induced by the flip of one of its two free spins. The precise characterization of this diffusive dynamics (whether normal, sub- or super-diffusive) requires further studies.

[17] R.G. Palmer, D.L. Stein, E. Abrahams and P.W. Anderson, Phys. Rev. Lett. **53**, 958 (1984).

[18] L. Viana and A.J. Bray, J. Phys. C **18**, 3037 (1985). M. Mézard and G. Parisi, Europhys. Lett. **3**, 1067 (1987). I. Kanter and H. Sompolinsky, Phys. Rev. Lett. **58**, 164 (1987).